Verification of Large Beam-Type Space Structures

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This paper describes the verification approach of large beam-type space structures. The proposed verification approach consists of two parts. The first is to remove the gravity effect on the tested substructure and to identify the on-orbit dynamic characteristics of the substructure by using the measurements of the ground test. A scaling law is also established to define the critical length of the structure that can be tested in 1 g field without incurring a buckling problem. The second part is to develop an adequate scaling law to extrapolate the dynamic characteristics of the prototype structure by using results from the substructure. The verification approaches are demonstrated on two typical structural configurations: the feed support structure of a wrap-rip antenna and a candidate Shuttle flight experiment. The results indicate that it is practical to verify the on-orbit dynamic characteristics of these structures by using the proposed approach.

Nomenclature

A = cross-sectional area of a simple beam
A_b, A_d, A_ℓ = cross-sectional area of batten, diagonal, and longeron, respectively
b = length of batten
C = corrected constant due to shear effect
d = length of diagonal
E = elastic modulus
g = gravitational acceleration
I = cross-sectional moment of inertia of a simple beam
I_b, I_d, I_ℓ = cross-sectional moment of inertia of batten, diagonal, and longeron, respectively
L = total length of a simple beam or a laced column
ℓ = length of longeron (i.e., bay length)
M = mass per unit length
m = vibration mode number
N = axial stretching force
N_{cr} = critical buckling stress of longeron
n = number of bays of a laced column
n_{cr} = critical buckling bay number
n_p = bay number of the prototype
n_{sb} = bay number of the substructure
W = static deformation
W_i = series coefficients determined by static deformation
$\gamma_b, \gamma_d, \gamma_\ell$ = density of batten, diagonal, and longeron, respectively
λ = critical gravitational multiplier
ν = Poisson's ration
ω_{gm}, ω_{0m} = natural frequency of the mth mode in 1 and 0 g fields, respectively
ω_p, ω_{sb} = natural frequency of the prototype and substructure, respectively

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Introduction

STRUCTURES to be used for future space application will be very large in size, such as space station or large deployable antenna systems.[1] These space structures may have dimensions on the order of 30–200 m. The major technical problem that must be overcome before large flexible structures can be utilized for future missions is to develop confidence in the predictions of their on-orbit dynamic characteristics. Current test methods are inadequate for such structures because of their service configurations and the effect of ground test environments. Methods must be developed to accurately predict on-orbit dynamic characteristics of large very flexible structures by utilizing ground test data obtained from either multiple supports, scale model testing, or substructure testing. A possible approach to this problem is addressed in this paper.

Since many large flexible space structures can be modeled as beams,[2] the generic structural element chosen for this investigation is a large space beam. The results obtained from analyzing a large space beam are applied to large multidimensional beam-type space structures, such as a typical feed support structure for a wrap-rib antenna[3] and the MAST, a deployable beam Shuttle flight experiment being planned by NASA as part of the Control of Flexible Structure (COFS) program.[4] The approach of this work is to perform a series of analytical investigations to examine the applicability of scale model ground testing for the determination of structural dynamic characteristics and to examine the applicability of testing a full-scale substructure in a 1 g environment. These analyses establish dimensionless parameters for verifying structural characteristics of large beam-type space structures and establish the limitations of these test methods for structural verification.

The verification approach presented in this paper consists of two parts. The first is to investigate the gravity effect on the dynamic characteristics of a large space beam. A closed-form solution for the dynamic response of a large space beam subjected to its own weight has been derived previously.[5] The results provide a better understanding of structural characteristics of a large space beam under gravity. In addition, the relationships for the natural frequencies in 1 and 0 g fields are formulated. This allows the identification of the on-orbit dynamic characteristics of large beam-type structures by utilizing the ground test data of such structures.

The second part of the verification approach is to develop scaling laws. A scaling law for the critical buckling length of large laced columns is established. This allows the selection of an adequate length of the structures for ground test. Another scaling law for the bay number of the structure with replicable bays is also developed. The results can be applied to extrapolate the dynamic characteristics of a large prototype structure by using the testing data of a substructure. In order to obtain more representative results, the shear effect is accounted for in developing this scaling law. Alternative approaches, such as suspending the system vertically, are also discussed in this work. Finally, the approaches developed in this work are demonstrated on both a typical feed support structure of a wrap-rib antenna and the MAST configuration. Numerical results from the NASTRAN code as well as the closed-form solution are presented.

Gravity Effect

The free vibration of a large space beam with simply supported ends subjected to its own weight (Fig. 1) has been investigated in Ref. 5. The results established the relationship of the natural frequencies in the 1 g field to those in a 0 g field. They are expressed by

$$\frac{\omega_{gm}}{\omega_{0m}} = \left[1 + \frac{NL^2}{m^2\pi^2 EI} + \frac{AW_i^2}{2I}\right]^{\frac{1}{2}}, \quad \text{for } i = m = 1,3,5,\ldots \tag{1a}$$

$$\frac{\omega_{gm}}{\omega_{0m}} = \left[1 + \frac{NL^2}{m^2\pi^2 EI}\right]^{\frac{1}{2}}, \quad \text{for } m = 2,4,6,\ldots \tag{1b}$$

where ω_{gm} is the natural frequency of the mth mode due to gravity effect, ω_{0m} the natural frequency of the mth mode in the 0 g environment, N the axial stretching force, L the beam length, E Young's modulus, I the cross-sectional moment of inertia, A the cross-sectional area, and W_i the series coefficient determined from the static deformation $W(x)$ due to its own weight,

$$W(x) = \sum_{i=1,3,5,\ldots} W_i \sin\frac{i\pi x}{L} \tag{2}$$

Equation (1) indicates that the natural frequencies of the symmetric modes ($m = 1,3,5,\ldots$) depend not only on the axial stretching force but also on the static deformation due to its own weight. However, the natural frequencies of the asymmetric modes ($m = 2,4,6,\ldots$) are not affected by the static deformation. It should be pointed out that the results shown in Eq. (1) are based on the linearized approach of the governing equation. The vibration amplitude is assumed to be relatively small compared to the static deformation due to its own weight in a 1 g field. For a large vibration amplitude, the nonlinear behavior of free vibration can be obtained from Ref. 5. The present paper will consider only small-amplitude vibration.

The dynamic characteristics of a vertically hanging beam (Fig. 2) subjected to gravity effect can be derived by using the energy method. The normalized frequency equation for all m can be expressed by

$$\frac{\omega_{gm}}{\omega_{0m}} = \left[1 + \frac{MgL^3}{2\pi^2 m^2 EI}\right]^{\frac{1}{2}} \tag{3}$$

where M is the mass per unit length. It should be pointed out that for the laced columns the mass M in Eq. (3) is the total mass of the structure divided by the total length of the structure.

Limitations of Ground Tests

The results discussed above allow the verification of structural characteristics of large beam-type structures in space by utilizing the ground test data of such structures. However, one of the limitations of the ground test for a very large flexible structure is the buckling of the structure due to its own weight. This kind of buckling problem will restrict the length of the structure tested in a 1 g environment. In order to define the critical buckling length of the structure in a 1 g field, a scaling law must be established.

Generally, the results of buckling analyses provide the eigenvalues and their corresponding buckling modes. The eigenvalue is the factor by which the prebuckling stresses are multiplied to produce buckling. Since the loading environment is designated as 1 g, the relationship between the structure length and the critical gravity multiplier (eigenvalue) must be established in order to define the critical buckling length of the structure in the designated 1 g field.

A typical buckling mode of a 20 bay structure subjected to a field 7.2 times Earth gravity is shown in Fig. 3. The geometric dimensions and material properties of this structure are obtained from Ref. 3 and are shown in Fig. 4. Shown in Fig. 3 is a local-type buckling mode of the top longerons. This occurs because the compressive stresses in the top longerons exceed the critical buckling stresses. Numerical results based on NASTRAN results, shown in Table 1, indicate that the critical buckling stress of the longeron N_{cr} is not significantly affected by the total length of the structure. This is because the critical load of local longeron buckling depends upon the length of the longerons, not upon the total length of the structure. Based on the assumption that the critical buckling stress of the longerons remains constant, it can be derived that the critical gravity multiplier is inversely proportional to the square of the bay number, if each bay of the structure is replicable. This can be expressed by

$$\lambda = (n_{cr}/n)^2 \tag{4}$$

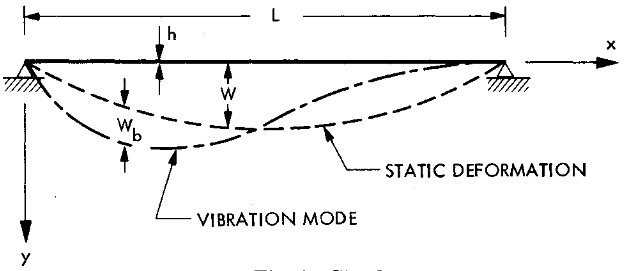


Fig. 1 Simply supported beam.

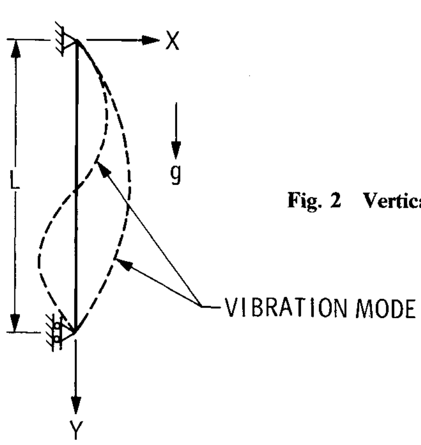


Fig. 2 Vertically hanging system.

Table 1 Critical buckling stress of longeron vs bay number

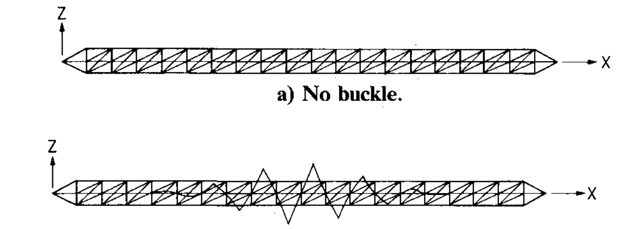
Bay no.	20	40	60	80	100
N_{cr}, ksi	1.44	1.41	1.40	1.39	1.39

Table 2 Predicted critical bay number of the feed support structure with different bay numbers

Bay no.	20	40	53	60	80
λ	7.20	1.75	0.992	0.772	0.432
n_{cr}	54	53	53	53	53

Table 3 Critical buckling bay number of a two-dimensional feed support structure

Bay no.	10	20	40
λ	21.70	5.20	1.28
n_{cr}	47	46	46



a) No buckle.

b) Buckled.

Fig. 3 Buckling mode of a 20 bay feed support structure (top view).

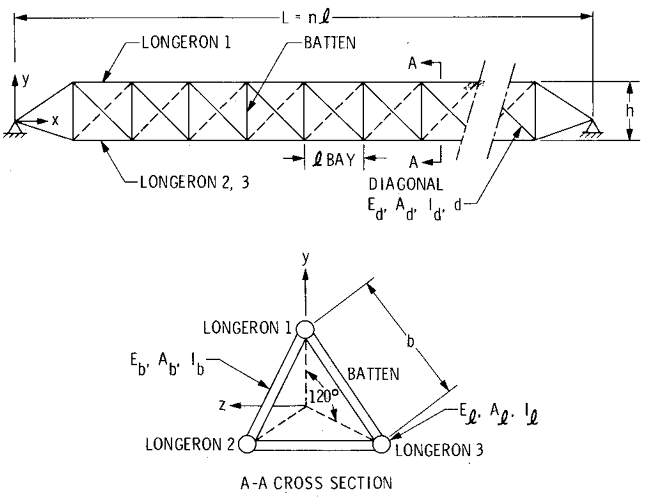


Fig. 4 Geometric configuration of the feed support structure.

where λ is the critical gravity multiplier of an n bay structure and n_{cr} the critical buckling bay number of the structure in a 1 g field. Table 2 shows the critical gravity multiplier, based on NASTRAN results, as a function of the bay number. Applying Eq. (4), the critical bay number of this structure can be predicted. These are also listed in Table 2. Satisfactory results are observed. In addition, it is noted that the lowest buckling mode of a two-dimensional 20 bay structure is a global lateral buckling (Fig. 5). Table 3 indicates that Eq. (4) is also valid for this kind of lateral buckling mode.

Scaling Law

Since the buckling problem limits the length of the structure tested on the ground and each bay of the structure is replicable, a proper approach to successfully conduct a ground test is to test the structure with a number of bays less than the critical number of bays. Therefore, a scaling law must be established in order to extrapolate nature frequencies of the prototype structure by using results from substructure testing.

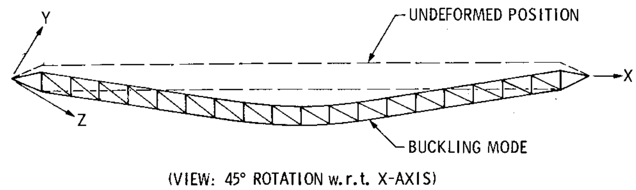


Fig. 5 Lateral buckling mode of a two-dimensional feed support structure.

This kind of scaling law will be particularly useful if the structural characteristics of the tested beams cannot be clearly defined in the analytical model, such as the joint effect of the deployable trusses.

It is known that the natural frequency of a uniform beam is inversely proportional to the square of the beam length. This is based on the assumption that the shear effect is negligible. However. Ref. 6 indicates that the effect of shear on the deflection is much greater for a laced column than for a solid beam. Hence, this kind of shear effect must be considered in large beam-type space structures, such as the typical feed support structure of a large antenna or the MAST.

It is noted that the effect of the shearing force reduces the critical buckling load of a laced column. This must be considered as the stiffness of the structure is decreased due to the action of shearing forces. In order to account for this effect in the vibration problem, the stiffness term in the frequency equation should be modified. This modified stiffness can be approximated from the buckling strength of a laced column. Following an approach similar to that used in Ref. 7, the modified stiffness EI_e of a triangular laced column as shown in Fig. 4 can be expressed by

$$EI_e = \frac{EI}{1+(C/n^2)} \tag{5}$$

where EI is the bending stiffness of the laced column (which can be approximately expressed by $EA_L b^2/2$), C/n^2 is the correction term due to the shear effect, and the constant C depends upon the structural geometry and vibration modes. For a triangular laced column, the constant C for the bending modes can be expressed by

$$C = \frac{2m^2\pi^2 EI}{3\ell^3 b^2}\left(\frac{d^3}{EA_d} + \frac{b^3}{EA_b}\right) \tag{6a}$$

where ℓ is the length of the longerons, d the length of the diagonals, b the length of the battens, and EA_d and EA_b the axial stiffnesses of the diagonals and the battens, respectively. Equation (6a) can be rewritten as

$$C = \frac{\pi^2 m^2 A_\ell}{3\ell^3}\left(\frac{d^3}{A_d} + \frac{b^3}{A_b}\right) \tag{6b}$$

Substituting the modified stiffness into the frequency equation of a beam subjected to lateral vibration, the scaling law can be expressed as

$$\frac{\omega_p}{\omega_{sb}} = \frac{n_{sb}^2}{n_p^2}\left[\frac{1+(C/n_{sb}^2)}{1+(C/n_p^2)}\right]^{\frac{1}{2}} \tag{7}$$

where ω_p is the natural frequency of the prototype structure, ω_{sb} the natural frequency of the substructure, n_p the bay number of the prototype structure, and n_{sb} the bay number of the substructure. It should be noted that the first part of the right-hand side of Eq. (7) accounts for pure bending and the second part accounts for the shear effect.

Table 4 Comparison of predicted natural frequencies for a 60 bay structure[a]

m	n	ω_m (NASTRAN), Hz	ω^b/ω_m^*	ω^s/ω_m^*
1	20	4.10	0.785	1.051
1	40	1.24	0.950	1.007
1	60	0.58		
2	20	10.67	0.577	1.070
2	40	4.00	0.864	1.016
2	60	2.05		
3	20	17.04	0.477	1.065
3	40	7.14	0.800	1.018
3	60	3.97		

[a] ω^b is the predicted natural frequency based on the assumption that the shear effect is negligible, ω^s the predicted natural frequency based on the scaling law which includes the shear effect, and ω_m^* the natural frequency of the mth mode for the 60 bay structure (NASTRAN results).

Table 5 Geometric dimensions and material properties of the feed support structure

Length of longerons ℓ	95 in.
Length of battens b	95 in.
Length of diagonals d	134 in.
Cross-sectional area	
Longerons A_ℓ	0.22 in.^{2}
Battens A_b	0.22 in.^{2}
Diagonals A_d	7.85×10^{-3} in.^{2}
Cross-sectional moment of inertia	
Longerons I_ℓ	0.11 in.^{4}
Battens I_b	0.11 in.^{4}
Diagonals I_d	4.91×10^{-6} in.^{4}
Mass density, $\gamma_\ell = \gamma_b = \gamma_d$	0.06 lb/in.^{3}
Elastic modulus, $E_\ell = E_b = E_d$	1.66×10^7 lb/in.^{2}
Poisson's ratio ν	0.3

Table 6 Measurements based on NASTRAN results

Measurements	Feed support structure (40 bay)	MAST (10 bay)
Max deformation, in.	14.64	0.0406
Max compressive stress, psi	8054	344
Max tensile stress, psi	4043	192
Natural frequencies in 1 g field, Hz		
$m = 1$	0.953	17.23
$m = 2$	3.086	47.91

Table 7 Comparison of natural frequency of large beam-type space structures, Hz[a]

Structure	Mode	ω_p	ω_e
Feed support			
structure	1	0.418	0.415
(60 bay)	2	1.554	1.523
Mast	1	0.767	0.715
(54 bay)	2	3.097	2.749

[a] ω_p is the predicted natural frequencies from verification approach and ω_e the NASTRAN results.

The scaling law of Eq. (7) is verified by using a two-dimensional feed support structure. The geometric dimensions and material properties of this structure are the same as those shown in Fig. 4. The constant C in Eq. (7) for a two-dimensional laced column can be obtained directly from Ref. 6. Both 20 and 40 bay laced columns are used to predict the natural frequencies of a 60 bay structure. Note that the structure will exhibit lateral buckling if the bay number exceeds 47, as shown in Table 3. The natural frequencies of these structures are calculated by using NASTRAN and are listed in Table 4. The comparison between the predicted natural frequencies of a 60 bay structure and those from NASTRAN results are also shown in Table 4. The results indicate that the effect of shear plays a significant role on extrapolating the natural frequencies of a longer laced column. It also shows that the scaling law based on Eq. (7) provides satisfactory results.

Verification Process

The results discussed above can be applied to verify the on-orbit dynamic characteristics of large beam-type space structures. The verification process can be summarized in the following steps:

1) Implementation of the buckling analysis for the structure subjected to its own weight provides the critical gravity multiplier (eigenvalue) and its corresponding buckling mode.

2) Application of the scaling law for the critical buckling length, as shown in Eq. (4), determines the critical buckling bay number of the structure in a 1 g field.

3) Selection of a structure with a bay number less than the critical bay number for ground testing provides substructure testing measurements in the 1 g environment, such as the static deformation, axial stresses, and natural frequencies.

4) Application of the frequency equation, as shown in Eq. (1) or (3), removes the gravity effect and determines the natural frequencies of the selected substructure in a 0 g field.

5) Application of the scaling law for bay number, as shown in Eq. (7), verifies the on-orbit natural frequencies of the prototype structure.

Applications

Two large beam-type space structures are examined. The first one is a typical feed support structure of a wrap-rib antenna, shown in Fig. 4. Its geometric dimensions and material properties are listed in Table 5. The results from the buckling analysis associated with the scaling law indicate that the structure will buckle due to its own weight if the bay number of this structure exceeds 54. In order to prevent the buckling problem, a 40 bay structure is proposed for the ground test. Since no real ground testing is anticipated in the example problem, the measurements of this 40 bay structure are assumed to be those obtained from NASTRAN results as listed in Table 6. Following steps 4 and 5 as discussed in the verification process, the on-orbit natural frequencies of a longer structure (such as 60 bay) can be determined and these are listed in Table 7 together with the direct NASTRAN results for comparison. A good agreement is observed.

The second space structure examined in this work is based on the MAST configuration being considered by NASA for a future flight experiment.[4] The material properties and geometric dimensions of the MAST are listed in Table 8. The full length of the prototype MAST is approximately 60 m (54 bays). However, a 10 bay MAST is proposed for the ground test because of the buckling limitation of the structure subjected to the gravitational environment. The ground test data of this 10 bay MAST, based on NASTRAN results, are also shown in Table 6. Following the verification process as discussed previously, the natural frequencies of this 54 bay MAST can be predicted and the results are also shown in Table 7. The higher discrepancy shown in this case is believed to be due to the smaller number of bays used in the ground test. The scaling factor due to shear effect is more accurate for a laced column with a large number of panels. For instance, if a 20 bay MAST could be tested in the 1 g field, better results could be achieved.

An alternate approach of verifying on-orbit dynamic characteristics of this MAST configuration is to test MAST substructure suspended vertically. The restriction of the MAST length, due to buckling caused by its own weight, is no longer

Table 8 Geometric dimensions and material properties of the MAST

Overall geometry	
Total length L	2380.86 in.
Length ℓ of each bay	44.09 in.
Diameter enclosing the MAST D	55.12 in.
Bay number	54
Cross section	
Longerons (3)	
Length ℓ	44.09 in.
Inside diameter of all longerons	0.55 in.
Outside diameter of top longerons	0.812 in.
Outside diameter of bottom longerons	0.763 in.
Diagonals (3)	
Length d	64.91 in.
Outside diameter (solid)	0.287 in.
Battens (3)	
Length (b)	47.64 in.
Inside diameter	0.25 in.
Outside diameter	0.328 in.
Material	
Graphite epoxy	
Young's modulus E	9.62×10^6 psi
Poisson's ratio ν	0.3
Mass	
Joints	0.787 lb
Specific weight density	
Longeron	0.07814 lb/in.^{3}
Diagonal	0.1604 lb/in.^{3}
Batten	0.05954 lb/in.^{3}

Table 9 Comparison of natural frequencies of a 20 bay MAST structure hanging vertically, Hz[a]

Mode	ω_0	ω_g^1	ω_g^2
1	4.883	4.888	4.887
2	16.524	16.531	16.528

[a] ω_0 is the natural frequencies in 0 g field (NASTRAN), ω_g^1 the natural frequencies in 1 g field (NASTRAN), and ω_g^2 the natural frequencies in 1 g field [Eq. (3)].

Table 10 Comparison of natural frequencies of the prototype MAST[a]

Mode	ω_p, Hz	ω_e, Hz	ω_p/ω_e
1	0.719	0.715	1.005
2	2.841	2.749	1.033

[a] ω_p is the predicted natural frequencies from verification approach and ω_e the NASTRAN results.

a major concern in the vertical suspension test. A 20 bay MAST is chosen for the vertical suspension approach. Results, as shown in Table 9, indicate that the gravity effect on the natural frequencies of a 20 bay MAST hanging vertically is insignificant. The predicted natural frequencies of the prototype MAST, based on a vertical suspended approach, are shown in Table 10. Better results are observed in this case.

Conclusions

An approach for the verification of a beam-type space structure has been described. The effect of gravity on the dynamic characteristics of both horizontally and vertically supported beams has been studied and the results are applied to identify the on-orbit dynamic characteristics of the structure tested on the ground. The natural frequencies of the full-size structure are extrapolated from those of the substructure by using scaling laws. The results indicate that, in order to accurately predict the natural frequencies of a laced column, the shear effect should be considered in this scaling law. NASTRAN analyses are implemented to verify the results based on the proposed verification approaches. Less than 2% errors are observed in verifying the first two on-orbit natural frequencies of both the typical feed support structure of a wrap-rib antenna and the MAST configuration.

Acknowledgment

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